



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 37

Put Call Parity & Arbitrage

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PUT CALL PARITY

	$t=0$	$t=T$	
PORTFOLIO A		$S_T < K$	$S_T > K$
BUY CALL	$-c$	0	$S_T - K$
INVEST	$-Ke^{(-rT)}$	K	K
TOTAL	$-c - Ke^{(-rT)}$	K	S_T



	$t=0$	$t=T$	
PORTFOLIO A		$S_T < K$	$S_T > K$
TOTAL	$-c - Ke^{(-rT)}$	K	S_T
PORTFOLIO B			
BUY STOCK	$-S_0$	S_T	S_T
BUY PUT	$-p$	$(K - S_T)$	0
TOTAL	$-S_0 - p$	K	S_T
$c + Ke^{(-rT)} = S_0 + p$			

PUT CALL PARITY

	$t=0$	$t=T$	
PORTFOLIO A		$S_T < K$	$S_T > K$
BUY CALL	$-c$	0	$S_T - K$
TOTAL	$-c$	0	$S_T - K$



	$t=0$	$t=T$	
PORTFOLIO A		$S_T < K$	$S_T > K$
TOTAL	$-c$	0	$S_T - K$
PORTFOLIO B			
BUY STOCK	$-S_0$	S_T	S_T
BUY PUT	$-p$	$(K - S_T)$	0
TOTAL	$-S_0 - p$	$K + 0$	$K + (S_T - K)$
$S_0 + p - c = Ke^{(-rT)}$			

ARBITRAGE: $S_0 + p < Ke^{-rT} + c$

	t=0	t=T	
		$S_T < K$	$S_T > K$
BORROW	$S_0 + p - c$	$-(S_0 + p - c)e^{(rT)}$	$-(S_0 + p - c)e^{(rT)}$
WRITE CALL	$+c$	0	$-(S_T - K)$
BUY PUT	$-p$	$K - S_T$	0
BUY STOCK	$-S_0$	S_T	S_T
TOTAL	0	$K - (S_0 + p - c)e^{(rT)}$	$K - (S_0 + p - c)e^{(rT)}$
RISKLESS PROFIT $= K - (S_0 + p - c)e^{(rT)} > 0$			

PUT CALL PARITY (DOLLAR RETURN)

	t=0	t=T	
PORTFOLIO A		$S_T < K$	$S_T > K$
BUY CALL	$-c$	0	$S_T - K$
INVEST	$-Ke^{(-rT)} - D_0$	$K + D_T$	$K + D_T$
TOTAL	$-c - Ke^{(-rT)} - D_0$	$K + D_T$	$S_T + D_T$



	t=0	t=T	
PORTFOLIO A		$S_T < K$	$S_T > K$
TOTAL	$-c - Ke^{(-rT)} - D_0$	$K + D_T$	$S_T + D_T$
PORTFOLIO B			
BUY STOCK	$-S_0$	$S_T + D_T$	$S_T + D_T$
BUY PUT	$-p$	$K - S_T$	0
TOTAL	$-S_0 - p$	$K + D_T$	$S_T + D_T$
$c + Ke^{(-rT)} = \underbrace{(S_0 - D_0)}_{\text{}} + p$			

PUT CALL PARITY (YIELD)

	t=0	t=T	
PORTFOLIO A		$S_T < K$	$S_T > K$
BUY CALL	-c	0	$S_T - K$
INVEST	$-Ke^{(-rT)}$	K	K
TOTAL	$-c - Ke^{(-rT)}$	K	S_T



	t=0	t=T	
PORTFOLIO A		$S_T < K$	$S_T > K$
TOTAL	$-c - Ke^{(-rT)}$	K	S_T
PORTFOLIO B			
BUY ASSET	$-S_0 e^{(-qT)}$	S_T	S_T
BUY PUT	-p	$K - S_T$	0
TOTAL	$-S_0 e^{(-qT)} - p$	K	S_T
$c + Ke^{(-rT)} = S_0 e^{(-qT)} + p$			

BOUNDS OF EUROPEAN CALLS

	t=0	t=T	
		$S_T < K$	$S_T > K$
BUY CALL	-c	0	$S_T - K$
SHORT STOCK	$+S_0$	$-S_T$	$-S_T$
INVEST	$-Ke^{(-rT)}$	K	K
TOTAL	$-c - Ke^{(-rT)} + S_0$	$K - S_T > 0$	0

NET CASH FLOW AT t=0: $+S_0 - c - Ke^{(-rT)} < 0$

or $c > S_0 - Ke^{(-rT)}$

UPPER BOUND: $c < S_0$

BOUNDS OF EUROPEAN PUTS

	t=0	t=T	
		$S_T < K$	$S_T > K$
BUY PUT	$-p$	$K - S_T$	0
BUY STOCK	$-S_0$	S_T	S_T
BORROW	$Ke^{(-rT)}$	$-K$	$-K$
TOTAL	$Ke^{(-rT)} - S_0 - p < 0$	0	$S_T - K > 0$

NET CASH FLOW AT $t=0$: $Ke^{(-rT)} - S_0 - p < 0$ or $p > Ke^{(-rT)} - S_0$

UPPER BOUND: $p < Ke^{(-rT)}$

BOUNDS: CALLS WITH DIVIDEND

	t=0	t=T	
		$S_T < K$	$S_T > K$
BUY CALL	$-c$	0	$S_T - K$
SHORT STOCK	$+S_0$	$-S_T - D_T$	$-S_T - D_T$
INVEST	$-Ke^{(-rT)} - D_0$	$K + D_T$	$K + D_T$
TOTAL	$-c - Ke^{(-rT)} + S_0 - D_0$	$K - S_T > 0$	0

NET CASH FLOW AT t=0: $S_0 - (c + Ke^{(-rT)} + D_0) < 0$ or $c > S_0 - D_0 - Ke^{(-rT)}$



EXAMPLE 1

- The spot price of silver is Rs 15,000 per kg. The storage and insurance costs etc are Rs 4,000 p.a. payable quarterly in advance. The premium on a six month European call with an exercise price of Rs 16,000 is Rs 2,400. If the risk free rate is 12% p.a., what is the premium on a corresponding European put option with the same exercise price and maturity (in Rs)?

TIME	(MONTHS)				0	3	6
STORAGE COSTS					1000	1000	0
INTEREST					0	0.03	0.06
DISCOUNT FACTOR					1	0.9704	0.9417645
PV OF STORAGE COSTS					1000	970.45	0
TOTAL PV					1970.45		
EXERCISE PRICE					16000		
PV OF EXERCISE PRICE					15068.2		
CURRENT STOCK PRICE					15000		
CALL PREMIUM					2400		
PUT PREMIUM					497.787		



EXAMPLE 2

- An investor sells a European call option with strike price of K and maturity T and buys a put with the same underlying, strike price and maturity. Describe the investor's position.



EXAMPLE 3

- Describe the terminal value of the following portfolio: a newly entered-into long forward contract on an asset and a long position in a European put option on the asset with the same maturity as the forward contract and a strike price that is equal to the forward price of the asset at the time the portfolio is set up.



AMERICAN OPTIONS

